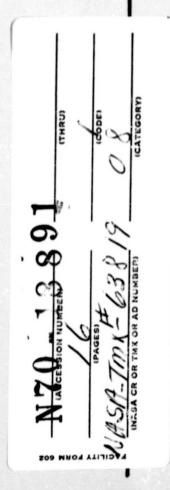# General Disclaimer

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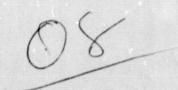# ASP
ARTIFICIAL SCIENTIFIC PROGRAMMING

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FEBRUARY 1970

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ASP

ARTIFICIAL SCIENTIFIC PROGRAMMING

by

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and
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INTRODUCTION

Artificial Scientific Programming (ASP) is an
application language designed to facilitate the computer
solution of a large class of scientifically oriented
problems. The salient feature of an application language
is the minimum effort required to program a computer
solution.

The ASP language is implemented on an IBM 360/91
with an MVT job scheduler. Input is accepted from cards,
a remote input terminal system RITS, or a question-answer
session using the 2250 display unit. The free form
statements are translated into a Fortran IV program,
which is optionally compiled and executed. Programs
generated by ASP are readable and flexible enough to
make basic alterations a simple process, because
general numerical algorithms frequently require modification
to solve a difficult problem.

In this report, the ASP language elements necessary
to solve a system of first order ordinary differential
equations are defined. A fifth order floating point
version of Nordsieck's method[1] is used to obtain
solutions of first order initial value problems.
Nordsieck incorporates iterative self starting, and

automatic revision of the step length into an efficient

general purpose method. The step length is chosen such

that a relative measure of the truncation error per

integration step is less than a specified bound. To

ensure the stability of the differential equations, it

is necessary to compute the spectral radius of the

Jacobian matrix[2]. Consequently, the assurance of stability

is left as a user supplied constraint on the step length.

ASP SYNTAX

The following conventions are adhered to in the description of the ASP language:

1. Statements are order independent unless explicitly stated otherwise, while expressions within a particular statement field are evaluated in the order they appear. Blanks are ignored except for headings.

2. Variable names, arithmetic expressions, and functions should be written following the normal Fortran conventions, with the restriction that names be no more than four characters.

3. A lower case e represents a Fortran arithmetic expression, c a floating point constant, k an integer constant, and v a variable name.

The first statement must be one of the three listed below:

    SOLVE; ODE:

    PROGRAM; DECK; ODE:

    PROGRAM; RITS; ODE:

For each of the above a Fortran program is generated which will solve a system of first order ordinary differential equations. If the first form is used the resulting Fortran program will be compiled and executed. Two and three will have the program punched on cards, or written on a disk so that it may be moved into a RITS

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library. The differential equations to be solved must immediately follow the SOLVE or PROGRAM request and are expressed as either

$$Y(1,1)=e_1; \quad Y(1,2)=e_2; \quad \text{---} \quad Y(1,n)=e_n:$$

or

$$F(1)=e_1; \quad F(2)=e_2; \quad \text{---} \quad F(n)=e_n:$$

where $Y(1,1)$ represents the first derivative of Y_1 with respect to T. The generated program is written to solve a system of equations with the standard nomenclature

$$\frac{dY}{dT} = F(T,Y)$$

where Y and F are vectors. If it is more convenient to type in the equations with names other than the standard, the following designation must appear

$$\text{VARIABLES:} \quad v_1; \quad v_2; \quad v_3:$$

For example, the system represented by

$$\frac{dP}{dR} = G(R,P)$$

may have the ASP definition

$$P(1,1)=e_1; \quad P(1,2)=e_2; \quad \text{---} \quad P(1,n)=e_n:$$

or

$$G(1)=e_1; \quad G(2)=e_2; \quad \text{---} \quad G(n)=e_n:$$

$$\text{VARIABLES:} \quad R; \quad P; \quad G:$$

If "computational" equations facilitate the writing of the differential equations, they are specified as

$$\text{EQUATIONS:} \quad v_1=e_1; \quad v_2=e_2; \quad \text{---} \quad v_n=e_n:$$

Summations are indicated by the ASP function

$$SIGMA(v=k_1,k_2; e)$$

To illustrate the way the EQUATION statement and the summation function are used consider the system of equations

$$Y_1 = Y_i/(Y_1+Y_2+Y_3)$$

This system may be written

$$Y(1,1)=Y(1)/R; \quad Y(1,2)=Y(2)/R; \quad Y(1,3)=Y(3)/R:$$

$$EQUATIONS: S=SIGMA(I=1,3; Y(I)); \quad R=SQRT(S):$$

SIGMA may only appear in assignment statements of the form $v=e_1+e_2*SIGMA(---)$, and its use is restricted to the EQUATION and DYNAMIC statements. The summand in the SIGMA function may contain nested sums such as the double sum

$$S2=SIGMA(I=1,10; A(I)*SIGMA(J=1,5; B(I,J)))$$

The size of arrays that appear in the program other than Y and F is transmitted by

$$ARRAYS: v_1(\underline{k}_1); \quad v_2(\underline{k}_2); \quad --- \quad v_n(\underline{k}_n):$$

where $\underline{k}$ denotes a vector of seven or less elements. A convenient way to initialize arrays is to use

$$DATA: v_1, v_2, \quad --- \quad v_n; \quad k_1*c_1, \quad k_2*c_2, \quad --- \quad k_n*c_n:$$

or

$$TABULAR: list; format:$$

The construction of these statements is best exemplified

by their ASP Fortran translations

$$\text{DATA } v_1, v_2, \text{ --- } v_n / k_1*c_1, k_2*c_2, \text{ --- } k_n*c_n /$$

and

$$\text{READ (5,xx) list}$$

$$\text{xx FORMAT (format)}$$

One procedure to initialize the matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

is to specify

ARRAYS: $A(3,3)$; $B(3,3)$:

DATA A: 3*1.0, 3*2.0, 3*3.0:

TABULAR: $((B(I,J),I=1,3),J=1,3)$; 3F5.1:

The data cards associated with each TABULAR statement follow the ASP program, and are arranged in the order requested. Statements used to represent parameters and Fortran arithmetic functions are of the form

PARAMETERS: $v_1=c_1$; $v_2=c_2$; --- $v_n=c_n$:

FUNCTIONS: $v_1(\underline{v}_1)=e_1$; $v_2(\underline{v}_2)=e_2$; --- $v_n(\underline{v}_n)=e_n$:

The interval of integration and the print increment are defined in

INTERVAL: c_1; c_2:   PRINT: c_3(default=1.0):

Backward integration is implicitly defined by $c_1 > c_2$. c_3 must be positive, and normally the solution will be

printed for $T=c_1$, c_1+c_3, --- c_1+kc_3, c_2. Initial
conditions are given with

INITIAL: $Y(1)=e_1$; $Y(2)=e_2$; --- $Y(n)=e_n$:
Frequently, it is necessary to have functions dependent
on the integrated values printed out along with the
solution. This is accomplished by including

DYNAMIC: $v_1=e_1$; $v_2=e_2$ --- $v_n=e_n$:
Another useful calculation associated with solutions
of differential equations is the evaluation of integrals.
Integration is specified in general by

INTEGRAL(v; e)
The limits of integration are assumed to be those in
the INTERVAL statement. For example to evaluate

$$A = \int \exp(-T) \ Y_1 \ dT$$
it is necessary to include

INTEGRAL(A; EXP($-T$)*Y(1)):
If the differential equations contain indeterminate
forms over the interval of integration the limiting
values are defined as

SINGULAR: $T=c$; $F(1)=e_1$; $F(2)=e_2$; --- $F(n)=e_n$:
The default value for a bound on the relative truncation
error per step is 10^{-5}. This may be changed by stating

PRECISION: k:
If $k \leq 5$ a single precision program is written, and double
precision if $k > 5$. When defining a double precision

program, it is necessary to use the corresponding double
precision Fortran functions if any appear in the ASP
expressions. If a fixed step length and no automatic
truncation error testing or step length modification
is desired

STEP: c_1; c_2:

must be included. The step length will have the value
c_1. The constant c_2 is optional and if inserted
signifies that truncation error testing is to be resumed
when the independent variable of integration is greater
than c_2. A heading to be printed on each page of output
is denoted

HEADING: 80 or less alphanumeric characters:

Comments may be inserted anywhere in the program and
are enclosed by double quotes

" any appropriate comment "

To have a condensed version of the output written on a
disk so that a listing on the remote terminal is possible
the statement

OUTPUT: TERMINAL:

must be included.

EXAMPLE3

For the first example a restricted three body
problem[3] (earth-moon-spaceship) is considered

$$\frac{d^2x}{dt^2} = x + 2\frac{dy}{dt} - a'(x + g) - a(x - g')$$

$$\frac{d^2y}{dt^2} = y - 2\frac{dx}{dt} - a'y - ay$$

with

$$g = 1/82.45, \quad g' = 1 - g$$

$$a = g / ((x - g')^2 + y^2), \quad a' = g' / ((x + g)^2 + y^2)$$

and initial values

$$t=0, \quad x_0=1.2, \quad x_0'=0, \quad y_0=0, \quad y_0'=-1.049935750983.$$

The solution x(t), y(t) is a closed orbit with period
t=6.192169331396. After a change of variable to rewrite
the equations as a first order system an ASP program for
this example is

    SOLVE; ODE:
    F(1)=U(3); F(2)=U(4);
    F(3)=U(1)+2.0*U(4)-AP*(U(1)+g)-A*(U(1)-GP);
    F(4)=U(2)-2.0*U(3)-U(2)*(AP+A):
    VARIABLES: T; U; F:
    EQUATIONS: A=G/DSQRT((U(1)-GP)**2+U(2)**2)**3;
    AP=GP/DSQRT((U(1)+G)**2+U(2)**2)**3:

PARAMETERS: G=1.21285627653312D-2;

GP=9.8787143723469D-1;

INITIAL: U(1)=1.2D+0; U(2)=0.0D+0; U(3)=0.0D+0;

U(4)=-1.04935750983;

PRECISION: 9; PRINT: 6.192169331396D-1;

INTERVAL: 0.0D+0; 6.192169331396;

HEADING: RESTRICTED 3-BODY PROBLEM:;

Two successive colons indicate the end of the ASP

program. The resulting solution is listed in Appendix B.

The next example is a solution of the regular

Coulomb function[4] which satisfies the differential

equation

$$\frac{d^2w}{d\rho^2} + (1 - 2\frac{\eta}{\rho} - \frac{L(L+1)}{\rho^2})w = 0$$

for L=0, η=1/2. An ASP program to solve this equation is

SOLVE; ODE:

    W(1,1)=W(2);

    W(1,2)=(-1.0+2.0*ETA/RHO-EL*(EL+1)/RHO**2)*W(1);

VARIABLES: RHO; W; F:

PARAMETERS: EL=0.0; ETA=0.5; CL=3.766858E-1;

INITIAL: W(1)=0.0; W(2)=CL;

SINGULAR: RHO=0.0; W(1,1)=CL; W(1,2)=CL;

HEADING: COULOMB FUNCTION:

INTERVAL: 0.0; 10.0; PRINT: 1.0:;

The solution from this program is also in Appendix B.

APPENDIX A

SUMMARY OF THE LANGUAGE ELEMENTS

ARRAYS: $v_1(\underline{k}_1)$; $v_2(\underline{k}_2)$; --- $v_n(\underline{k}_n)$:

DYNAMIC: $v_1=e_1$; $v_2=e_2$; --- $v_n=e_n$:

EQUATIONS: $v_1=e_1$; $v_2=e_2$; --- $v_n=e_n$:

FUNCTIONS: $v_1(\underline{v}_1)=e_1$; $v_2(\underline{v}_2)=e_2$; --- $v_n(\underline{v}_n)=e_n$:

HEADING: 80 or less alphanumeric characters :

INITIAL: $Y(1)=e_1$; $Y(2)=e_2$; --- $Y(n)=e_n$:

INTEGRAL(v; e):

INTERVAL: c_1; c_2:

OUTPUT: TERMINAL:

PARAMETERS: $v_1=c_1$; $v_2=c_2$; --- $v_n=c_n$:

PRECISION: k:

PRINT: c:

SIGMA($v=k_1,k_2$; e)

STEP: c_1; c_2:

TABULAR: list; format:

DATA: v_1, v_2, --- v_n; k_1*c_1, k_2*c_2, --- k_n*c_n:

VARIABLES: v_1; v_2; v_3:

" any comment "

PROGRAM OUTPUT

```
ASP (ODE-A) PROGRAM OUTPUT       RESTRICTED 3 BODY PROBLEM

PARAMETERS
G= 1.212856276531D-02       GP= 9.87871437234690-01

 T                         U(I), I=1,NE OR U(I)/F(I), I=1,NE
0.0                        1.2000000000000D 00   0.0
6.19216933139600-01        9.03563085324600-01  -5.17369054287030-01
1.23843338662792D 00       3.16989084586850-01  -3.92490278842450-01
1.85765079941880 00       -4.30887914326710-01  -5.87673181741330-01
2.47686773255584D 00      -9.99890120778920-01  -5.56156627405070-01
3.09608466569800 00       -1.26245435439300 00   7.69273074603150-09
3.71530159883760 00       -9.99890139246000-01   5.56156657350150-01
4.33451853197720 00       -4.30887939415940-01   5.87673263879860-01
4.95373546651680 00        3.16989045394120-01   3.92490166596410-01
5.57295239825640 00        9.03563077246120-01   5.17369017336760-01
6.19216933139600 00        1.20000003071600 00  -3.35859127519350-08


-INTEGRATION STEPS = 2776-
-TRUNCATION ERROR / STEP LESS THAN 1.0D-09
```

```
ASP (ODE-A) PROGRAM OUTPUT       COULOMB FUNCTION

PARAMETERS
EL= 0.0              ETA= 5.000000E-01
CL= 3.766858E-01

 RHO             W(I), I=1,NE OR W(I)/F(I), I=1,NE
0.0             0.0              3.766858E-01
1.000000E 00    5.166003E-01     5.929234E-01
2.000000E 00    1.021117E 00     3.296018E-01
3.000000E 00    1.043202E 00    -3.169894E-01
4.000000E 00    4.192396E-01    -8.667134E-01
5.000000E 00   -4.904488E-01    -8.331279E-01
6.000000E 00   -1.028593E 00    -1.643893E-01
7.000000E 00   -7.674270E-01     6.531553E-01
8.000000E 00    1.035077E-01     9.621581E-01
9.000000E 00    8.880071E-01     4.885538E-01
1.000000E 01    9.391693E-01    -3.957638E-01


-INTEGRATION STEPS = 122-
-TRUNCATION ERROR / STEP LESS THAN 1.0E-05
```

APPENDIX C

JCL AND DECK SETUP

job card

```
//JOBLIB DD DSN=SYS1.SNOBOL,DISP=SHR
// EXEC ASP
//ASP.INPUT DD *
```

ASP program

```
/*
//GO.DATA5 DD *
```

data cards specified by TABULAR statements
arranged in the order requested

REFERENCES

1. A. Nordsieck, "Numerical Integration of Ordinary Differential Equations," Mathematics of Computation, v. 16, 1962, p.22-49.

2. H. R. Lewis, Jr. and E. J. Stovall, Jr. "Comments on a Floating Point Version of Nordsieck's Scheme for the Numerical Integration of Differential Equations," Mathematics of Computation, v. 21, 1967, p. 157-161.

3. Roland Bulirsch and Josef Stoer, "Numerical Treatment of Ordinary Differential Equations by Extrapolation Methods," Numerische Mathematik, v. 8, 1966, p. 1-13.

4. Milton Abramowitz, "Coulomb Wave Functions," in Handbook of Mathematical Functions, fifth edition, U. S. Dept. of Commerece National Bureau of Standards Applied Mathematics Series 55, 1965, p. 537-554.